USA Mathematical Talent Search

Round 2 Grading Rubric

Year 37 — Academic Year 2025–2026

www.usamts.org

GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.
4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.
5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/2/37:

Award **5 points** for the correct configuration of entries. No justification is required. Withhold **1 point** for each entry that is incorrect (e.g., wrong number, blank instead of number).

Problem 2/2/37:

Note: Since the example of $n = 6$ is given in the problem statement, the entire problem is about showing that otherwise n is *amazing* if and only if n has at least three distinct prime factors.

1 point: Student shows that $n = p^e$ is not amazing.

2 points: Student shows that (excluding $n = 6$) if n has exactly two distinct prime factors, then n is not amazing. Award **1 point** for significant constructive progress towards this result. Withhold **1 point** if the student does not explain why $n = 6$ is the only exception.

2 points: Student shows that if n has at least three distinct prime factors, then n is amazing. Award **1 point** for significant constructive progress towards this result.

Note: Award a total score of **1 point** if the student writes the correct answer with little or no explanation.

Problem 3/2/37:

5 points: Student finds a pair of functions f and g that satisfies the requirements of the problem and it is clear that the three equations (e.g., $f(g(g(x))) = 3x$) are satisfied. For the pair of functions in the official solution, it is clear from the definitions of f and g that the three equations are satisfied, so students don't need to check algebraically that the functions work to get full credit. If the student comes up with a different pair of functions and it is not clear that the equations are satisfied, withhold at least **1 point**.

Note: Students who claimed that no pair of functions exists typically received a score of **0 points**. However, we awarded **1 point** if there was significant constructive progress. For example, if the student recognized a useful relationship such as $g(3x) = 5g(x)$, then the student received **1 point**.

Problem 4/2/37:

Note: Award a total score of **1 point** if the student provides both correct answers (not just one!) with little or no explanation.

Synthetic solution:

1 point: Student obtains a meaningful result, such as a useful pair of similar triangles, along with diagrams as needed.

2 points: Student shows that $PD = \sqrt{2}$. Award **1 point** of partial credit for a significant intermediate length result, such as $DQ = \sqrt{2}$.

2 points: Student shows that $PC = 2$. Award **1 point** of partial credit for a significant intermediate length result, such as $DS = \sqrt{2}$.

Note: Withhold **1 point** if the student doesn't include sufficient diagrams.

Complex number solution:

1 point: Student sets up the problem using complex numbers by placing the heptagon on the unit circle and obtaining expressions for the seven vertices using ω.

1 point: Student finds an expression for p in terms of ω and t, such as $p = -\omega^6 - \omega^5 + t\omega^4$.

1 point: Student solves for t, yielding an expression for p only in terms of ω, such as $p = \omega^2 - \omega^5$.

1 point: Student computes $PC = 2$.

1 point: Student computes $PD = \sqrt{2}$.

Note: Many calculators do approximations and thus cannot be used to obtain exact answers. Students did not receive credit for steps using calculators unless they used a symbolic calculator and showed their inputs and outputs.

Note: Students using the result that $8c^3 - 4c^2 - 4c + 1$ is the minimal polynomial for $\cos\left(\frac{\pi}{7}\right)$ needed to prove this result to earn full credit.

Problem 5/2/37:

Graph theory solutions:

1 point: Student proves that there is a complete graph on 100 vertices (i.e., a tournament).

2 points: Student proves that the tournament can be partitioned into strongly connected components such that a strongly connected component of size at least 10 contains a 10-cycle.

1 point: Student proves that the condensation of the strongly connected components creates a transitive subgraph.

1 point: Student shows that if the tournament does not have a 10-cycle, then the transitive subgraph must be of length at least 10.

Solutions not explicitly using graph theory:

1 point: Student proves the Claim: "Either there exists a team that draws against at least 10 other teams, or there exists a set of 100 teams, no two of which draw against each other."

1 point: Student proves Lemma 1: "Suppose that a round-robin tournament with n teams and no draws has no k-cycles for any k. Then the teams can be numbered from 1 to n such that team i beats team j for all $i < j$."

1 point: Student proves Lemma 2: "In a round-robin tournament without draws, suppose $(t_1, \ldots, t_k)$ is a cycle of maximal length. Then for any team $u \neq t_1, \ldots, t_k$, either u

beats t_i for all i, or u loses to t_i for all i."

1 point: Student proves Lemma 3: "For any round-robin tournament without draws, the teams can be partitioned into sets $S_1, \dots, S_m$ which satisfy the following properties:

1. For all $i < j$, if $t \in S_i$ and $u \in S_j$, then t beats u.
2. For each i, either $|S_i| = 1$ or all the teams in S_i can be arranged in a cycle."

1 point: Student proves Lemma 4: "Suppose a round-robin tournament without draws has a k-cycle. Then the tournament also has an ℓ-cycle for each ℓ such that $3 \leq \ell \leq k$."