USA Mathematical Talent Search

Round 1 Grading Rubric

Year 37 — Academic Year 2025–2026

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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.

4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.

5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/1/37:

Award **5 points** for the correct configuration of entries. No justification is required. Withhold **1 point** for each entry that is incorrect (e.g., wrong number, blank instead of number, number instead of blank).

If the solution is correct except that it contains 0's instead of blank entries, award **4 points**.

Problem 2/1/37:

1 point: Student sets up the problem algebraically and captures the key information in the problem. Recognizing that $a = 10^k \cdot x$ and $b = 10^k \cdot y$ and $x + y = xy$ is sufficient to earn this point.

1 point: Student does useful algebraic manipulations to obtain $(a - 10^k)(b - 10^k) = 10^{2k}$ or a similarly helpful equation and performs a meaningful analysis of this equation (e.g., recognizing that $x > 1$ and $y > 1$, neither a nor b contains a 0 in their decimal representations).

1 point: Student shows that

$$x = \frac{10^k + 2^{2k}}{10^k} = 1 + \left(\frac{2}{5}\right)^k,$$
$$y = \frac{10^k + 5^{2k}}{10^k} = 1 + \left(\frac{5}{2}\right)^k.$$

1 point: Student recognizes that $k = 1$ and $k = 2$ provide valid solutions, and student finds the corresponding solutions.

1 point: Student explains why there are no solutions for $k \geq 3$.

Note: If the student doesn't find both solutions or finds an incorrect solution, award at most **4 points**.

Note: If the student finds both solutions, award a total score of **1 point** if there is no explanation or the student just verifies that the solutions work. Award a total score of **1 point** if the student finds one solution and verifies that it works, but do not award any credit if the final answer is not fully correct and the student provides no explanation.

Note: The answer should be considered acceptable if the student makes it clear what are the numbers in each pair. For example, we don't care about ordered vs. unordered pairs. Don't dock a point if the student writes the final answer in fraction form, though decimal form is more appropriate given the theme of the problem.

Problem 3/1/37:

Note: Part (a) is worth **2 points** and part (b) is worth **3 points**.

Note: Since the problem has yes/no answers, do not award any credit just for the correct answers to either or both parts.

Part (a):

2 points: Student accurately applies the modifications to the pair $(0, 0)$ to attain $\left(2025, \frac{1}{2025}\right)$.

Award **1 point** of partial credit for incomplete solutions that achieve either of the following types of significant constructive progress:

(1) Student shows how the given modifications can be combined to create a more useful modification, such as proving that if (a, b) is attainable, then $(a, b+2)$ and $(a+2, b)$ are attainable.

(2) Student shows how it is possible to achieve $\left(1, \frac{1}{2025}\right)$ or $\left(3, \frac{1}{3}\right)$.

Note: Award **1 point** for this part if the student shows how to go from $\left(2025, \frac{1}{2025}\right)$ to $(0, 0)$, but not the desired direction.

Part (b):

1 point: Student proposes a reasonable way to calculate the parity of the pairs, such as for a given pair $\left(\frac{a_1}{a_2}, \frac{b_1}{b_2}\right)$ considering the parity of the value $|a_1 b_2 - a_2 b_1|$, and recognizes that the modifications are invariant with respect to parity. It is not enough for the student simply to claim that parity is relevant; they also need to consider the invariance with respect to parity to receive credit.

1 point: Student shows that parity is preserved for all four of the given modifications.

1 point: Student shows that the parities of $(0, 1)$ and $\left(2025, \frac{1}{2025}\right)$ are different and concludes that it is impossible to go from $(0, 1)$ to $\left(2025, \frac{1}{2025}\right)$.

Note: Unless the solution captures points in the rubric discussed above, no credit should be awarded for showing that it is impossible to go from $(0, 1)$ to $(0, 0)$, since perhaps there is some other way to go from $(0, 1)$ to $\left(2025, \frac{1}{2025}\right)$.

Problem 4/1/37:

2 points: Student proves the lemma about convex hexagons in the official solution or mentions this result with a valid citation. Withhold **1 point** if the student states the lemma without a proof or citation.

3 points: Student applies the lemma to prove the problem statement. Award these points as follows:

1 point: Student recognizes that we can select a starry tuple (A, B, C, D, E, F) which minimizes the size of the set T, the points of S in the interior of hexagon $AFBDCE$, proposes that we show that $T = \emptyset$, and makes some reasonable effort to develop this argument. This aspect of the solution is often framed in terms of the idea of "minimality" or "point swapping."

1 point: Student rigorously analyzes the case in which X is in $ABC \cap EAF$. A rigorous analysis includes showing that the convexity of the hexagon is preserved and that $XBC \cap DEF$ is inside $ABC \cap DEF$.

1 point: Student rigorously analyzes the case in which X is in AFB. A rigorous analysis includes showing that the convexity of the hexagon is preserved and being careful about the intersection of regions and where points in S might be. The supplemental explanation at https://files.usamts.org/y37r1p4.html discusses the importance of this case.

Note: Award a total score of **2 points** if the student gets at the idea of minimality and has a non-rigorous discussion of both the X is in $ABC \cap EAF$ and X is in AFB cases. Award a total score of **3 points** if the student also makes a reasonable attempt to address why the hexagons are convex, even if the justification of convexity is not fully rigorous.

Note: Withhold **1 point** if the student didn't include a diagram and a diagram would have been helpful. If significant parts of a solution are incomprehensible due to the absence of diagrams, only award credit for the parts you are able to understand with reasonable effort.

Problem 5/1/37:

Note: A student must do two things to achieve a complete and correct solution. First, a student must provide a valid construction of a k-good coloring with $k = \frac{n(n+1)}{2}$ for a general $n^2 \times n^2$ grid. Second, a student must show that $k = \frac{n(n+1)}{2}$ is the upper bound. The general construction is worth **2 points** and the proof of the bound is worth **3 points**.

2 points: Student provides a valid construction of a k-good coloring with $k = \frac{n(n+1)}{2}$ for a general $n^2 \times n^2$ grid. These points will often be divided as follows (**1 point each**):

(1) Student provides a diagram of a valid construction with $k = \frac{n(n+1)}{2}$ for $n \geq 3$.

(2) Student provides a written explanation of the general construction for any n.

Note: The written explanation can be pretty challenging, so cut students some slack here if there is a diagram. Award **2 points** as long as it is reasonably clear from the diagram and written explanation how to do the construction for higher values of n. On the other hand, if the student doesn't include a diagram, the written explanation better be crystal clear for the student to earn **2 points** for the "construction" part of the solution.

Note: If the student's construction works for even n or odd n, but not both, award **1 point** for the "construction" part of the solution.

3 points: Student proves that $k = \frac{n(n+1)}{2}$ is the upper bound. The details of the student proofs may vary, so you might be making some judgment calls about the severity of a gap in the explanation when awarding partial credit. For the official solution, the points are divided as follows:

1 point: Student shows that if r is the number of rows and c is the number of columns of a given color that have at least k cells of that color, then there must be $r + c \geq 2n$.

1 point: Student shows that there are $n^4 - n^3 \leq (r + c)(n^2 - k) + (n^2 - r)(n^2 - c)$ other-colored cells in the grid or a similarly helpful result.

1 point: Student uses these results to show that $k \leq \frac{n(n+1)}{2}$.

Note: If the student provides the correct answer of $k = \frac{n(n+1)}{2}$ with minimal or no explanation, award a total score of **1 point**.

Note: Submissions with the incorrect answer of $k = n$ will typically receive 0 points.