USA Mathematical Talent Search
Round 2 Grading Rubric
Year 35 — Academic Year 2023–2024
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.

4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.

5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/2/35:

Award **5 points** for the correct configuration of entries. No justification is required. Withhold **1 point** for each entry that is missing or incorrect.

Problem 2/2/35:

Note: We only need to know which number of distinct digits gives us the most possible codes, so it is fine if the student uses bounds rather than computing the exact number of codes.

Note: For students who compute the exact number of codes, award **1 point** each for the counts for each of 2 through 6 distinct digits. Give a maximum of **4 points** if the student doesn't do the 1-digit case.

Note: If a student consistently makes the same minor error when doing the counting, give a reasonable amount of partial credit depending on the severity of the counting error.

Note: If a student overlooks that it is known which digits are smudged, award at most **3 points**.

Problem 3/2/35:

3 points: Student proves that a similar "balance relationship" holds for a triangular grid of side length 4. Award partial credit as appropriate for significant constructive progress, for example **1 point** for useful thinking mod 3 and an additional **1 point** for the array at the top of the second page of the official solution. For students who use casework, award **3 points** if all cases are covered, and award appropriate partial credit depending on which cases are missing.

2 points: Student extends the balance relationship for a triangular grid of side length 4 to a triangular grid of side length 10.

Note: Quite a few students wrote computer programs. The usual guidelines for programming solutions apply, including the need to include the code and reasonable explanation of what the program is doing to earn **5 points**. Make sure the program actually does what the student says it's doing! Award partial credit for programming solutions if there is significant constructive progress, but a conceptual error in how the student is thinking about the problem or an error in the code that can be fixed.

Note: Some students solved the problem using binomial coefficients. For these solutions, award **1 point** for using the recurrence relation (only if used to set up some type of recurrence, as it's otherwise just a restatement of the problem condition). Award **2 points** for the binomial coefficient formula, and the final **2 points** for evaluating the binomial coefficients mod 3. Deduct **1 point** if the student just uses Pascal's triangle without justifying why sign can be ignored.

Problem 4/2/35:

Note: Comment on configuration issues, but don't deduct points for this.

1 point: Student draws a helpful diagram. If a solution would benefit from a diagram, but no diagram is included, award a total score of at most **4 points**.

1 point: Student shows that $IX = IY$. (This intermediate result is reasonably common across a variety of solution methods.) For students using Official solution 2, the equivalent useful result is that the midpoint of $\overline{XY}$ is the foot of the altitude from I to $\overline{XY}$.

Official solution 1: Award **2 points** for recognizing that

$$\text{Pow}_{(AEF)}X - \text{Pow}_{(DEF)}X = \text{Pow}_{(DEF)}(Y) - \text{Pow}_{(AEF)}(Y).$$

Award **1 point** of partial credit out of these **2 points** if the student does at least one meaningful application of Power of a Point, and tries to use the result in a potentially useful way.

Award the final **1 point** for recognizing that linearity of power gives us the desired result that $\overline{EF}$ bisects $\overline{XY}$.

Official solution 2: Award **3 points** for using I's Simson Line with respect to triangle AXY to recognize that E, F, and M are collinear, giving us that $\overline{EF}$ bisects $\overline{XY}$.

Problem 5/2/35:

2 points: Student comes up with the four constructions in (i) in the official solution, or an equivalent set of constructions. Award **1 point** of partial credit if the student comes up with any of the constructions in (i). Do not award credit for a case with specific numerical values, such as $(m, n) = (2, 3)$, unless there is meaningful progress towards a generalization. Additionally, do not award credit if only the $m = n$ case is covered, since this is not a start to a likely solution.

1 point: Student shows (iii), that if (m, n) works, then $(m + 2, n + 2)$ also works.

2 points: Student shows that all other pairs (m, n) don't work. Award partial credit for significant constructive progress towards this result, such as recognizing that if (m, n) works, M must contain an even number of elements (see official solution).

Note: There are other ways of describing the final answer, such as "$n - m \equiv 0 \pmod 4$ or $m + n \equiv 1 \pmod 4$," so check to see if the student's answer is equivalent to what we have in the official solution.

Note: Award **1 point** for the correct answer with no explanation or incorrect logic.