USA Mathematical Talent Search
Round 2 Grading Rubric
Year 31 — Academic Year 2019–2020
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to merit **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/2/31:

Award **5 points** for the correct configuration of numbers. Withhold **1 point** for each incorrect entry.

Problem 2/2/31:

4 points: Student uses complementary counting or another strategy to determine the number of citrus grids. For students whose work is similar to the official solution, these points should be awarded as follows. This breakdown should serve as a rough guideline for other approaches, but please use your judgment when awarding points.

To get **1 point**, it is sufficient to show the 9 possible permutations of the numbers (ignoring color), which are grids A through I in the official solution. Award an additional point (**2 points total**) if the student makes a useful observation based on the permutations. An example of such an observation is that if Grids A and B have the same coloring, then cells 1, 2, and 3 must have the same color, cells 4, 5, and 6 must have the same color, and cells 7, 8, and 9 must have the same color. Award an additional point (**3 points total**) if the student shows that there are four types of coloring schemes for non-citrus grids; these are grids W, X, Y, and Z in the official solution. Award an additional point (**4 points total**) if the student determines that there are 26 non-citrus grids.

1 point: Student recognizes that (1) there are 512 total grids and (2) we need to subtract the number of non-citrus grids from the total number of grids to determine that the probability Sophia receives a citrus grid is $\frac{243}{256}$. Neither (1) nor (2) by itself is sufficient to receive this point.

Note: If the student obtains the correct answer, but does not provide any explanation, they should receive a score of **1 point** on this problem.

Note: Even though we are basically giving **1 point** for the answer, don't double-penalize students if they make some error counting the number of citrus (or non-citrus) grids. In other words, if a student has a minor error in the counting, don't penalize students for both the counting error and the incorrect answer; they should receive **4 points**.

Note: We deducted **1 point** if a student mixed up the definitions of *citrus* and *non-citrus*.

Note: We deducted **2 points** if a student missed the diagonal cases Y and Z (**1 point** for each case).

Note: We deducted **2 points** if a student incorrectly applied cases Y and Z to only the case in which a main diagonal was one color and all the rest were the other color.

Note: We deducted **1 point** if a student asserted, without proof, that if there is one lime in each row and each column, the grid is not citrus. This is correct for 3×3 but not for larger grids.

Problem 3/2/31:

1 point: Student recognizes that $a(cm + d) - c(am + b) = ad - bc$.

2 points: Student shows that it is possible to have $|ad - bc| = 2019k$ for all positive integers k. Award **1 point** for proposing a fruitful quadruple that works and award **1 point** for showing that (1) the quadruple is fruitful and (2) the quadruple gives us $|ad-bc| = 2019k$.

2 points: Student shows that we cannot have $|ad - bc| = 0$. Award **1 point** for significant constructive progress towards this result. An example of significant constructive progress is showing that $\gcd(am + b, cm + d)$ is a multiple of $em + f$ (see official solution). Pointing out that $ad - bc = 0$ is equivalent to $\frac{a}{b} = \frac{c}{d}$ is **not** sufficient to receive credit.

Problem 4/2/31:

Note: One reason we asked for the numerical answer was to make it easier to see if there is an error somewhere in the student's work. If a student gave $\binom{50}{3}^2$, we did not deduct points. If a student correctly calculated an ugly sum (e.g., adding the daily totals of the number of candied cherries Princess Pear receives), we gave **5 points** if the explanation was clear. If such a sum was calculated incorrectly, we typically gave at most **3 points** unless there was a visible trivial error, in which case we awarded **4 points**. If the explanation was weak, we awarded fewer points.

Note: Some students wrote computer code. Please see the general guidelines for computer-based solutions on the first page of the grading criteria. Students need to include their code with enough explanation so that you can follow their work. If a student used Wolfram Alpha or other software, they need to show their inputs and outputs. Otherwise, do not award credit for the corresponding steps. The entire student solution should be included in the printout; if a student includes a link to additional work, you do not need to go to the linked website or award credit for the additional work.

Note: Relatively few students did a bijection solution, so we've only included a rubric for the algebra solution.

Note: The rubric below uses the notation in the official solution.

2 points: Student recognizes that in the first 100 days, Princess Pear receives

$$2\sum_{n=54}^{100}\sum_{x=1}^{n-53}\binom{n-51-x}{2}\binom{50-x}{2}$$

candied cherries. Award **1 point** of partial credit for a correct analysis of day 54 or for other significant constructive progress towards this result.

1 point: Student recognizes that on day 100, Princess Pear receives

$$\sum_{t=3}^{50}\binom{t-1}{2}^2$$

candied cherries.

1 point: Student constructively uses reindexing to make at least one of the summation expressions easier to compute.

1 point: Student uses the Hockey-Stick Identity or another method to determine that Princess Pear receives $384,160,000$ candied cherries.

Note: We typically awarded at most **3 points** if a student had a meaningful error in a combinatorial expression, including off-by-one errors.

Note: We deducted **1 additional point** if the student included jesters going up to height 100 instead of height n on day n.

Note: Award **1 point** for the correct answer if the student does not provide any explanation.

Problem 5/2/31:

Note: To get full credit, students needed to have at least the same level of detail as the official solution. If there was a key step that was not justified, we deducted points accordingly.

1 point: Student draws a reasonable figure based on the given information. This typically also applied to coordinate-based solutions, since most of those solutions also included geometric arguments for which a figure was (or would have been) useful.

1 point: Student recognizes that $\triangle DEF$ and $\triangle I_A I_B I_C$ are homothetic.

1 point: Student recognizes that O is the nine-point center of $\triangle I_A I_B I_C$ or makes other equivalent constructive progress beyond the previous step.

1 point: Student recognizes that if $AO = AH$, then $\angle A = 60°$ (the lemma in the Math Jam transcript) or makes other equivalent constructive progress beyond the previous step.

1 point: Student completes the proof (e.g., by applying the lemma to $\triangle I_A I_B I_C$).