USA Mathematical Talent Search

Round 3 Grading Rubric

Year 27 — Academic Year 2015–2016

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/3/27:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each number that is placed incorrectly.

Problem 2/3/27:

Award **5 points** if the student describes a strategy that will allow Fames to correctly announce that he has earned at least one more point, and the student provides a clear and convincing explanation. Partial credit may be awarded depending on the quality of the student's explanation.

Award a total of **2 points** if the students describes a strategy that will allow Fames to correctly announce that he has earned at least one more point, but the student does not explain why the strategy will work.

Award a total of **1 point** if the student describes a strategy that will allow Fames to eventually earn at least one more point with at least some justification.

A completely incorrect strategy is worth **0 points**.

Problem 3/3/27:

Award **1 point** if the student states that the maximum value of $\frac{a_n}{n} = \frac{1}{2}$ and shows that there is a value of n such that $\frac{a_n}{n} = \frac{1}{2}$.

Award **1 point** if the student obtains a useful expression for $\nu_p(n!)$, which is the maximum power of p that divides $n!$. One way to get this point is by applying Legendre's formula.

Award **1 point** if the student applies obtains the inequality

$$a_n < \frac{n}{k} \sum_{j=1}^{\infty} \frac{1}{p^j}.$$

In this inequality, $n = p^k m$, where p is some prime dividing n and m is relatively prime to p. This point should also be awarded if the student makes equivalent constructive progress beyond finding the expression for $\nu_p(n!)$.

Award **1 point** if the student simplifies the above inequality to the inequality

$$\frac{a_n}{n} < \frac{1}{k(p-1)},$$

or if the proof is otherwise almost complete.

Award **1 point** if the student completes the proof by reasoning that for all $n > 2$, $\frac{a_n}{n} < \frac{1}{2}$.

Problem 4/3/27:

Note: Student solutions varied considerably. Award full credit (**5 points**) for any correct and complete solution.

Partial credit should be awarded as follows:

Award **1 point** if the student draws a good diagram that shows the relevant information in the problem.

Award **1 point** if the student does something else meaningful (e.g., applies the Angle Bisector Theorem, identifies similar triangles, creates the foundation for a solution using coordinate geometry).

To get more points, it should be clear how what the student is doing can lead to a correct solution. Award a total of **3 points** if there is additional significant constructive progress, and award a total of **4 points** if the solution is almost complete or if the solution is complete with some weaknesses in the justification.

For students whose method followed the official solution, award **3 points** if the student finds (with justification) that $\frac{MP}{AC} = \frac{DM}{DC}$. Award **4 points** if the student applies the fact that if $\frac{a}{b} = \frac{c}{d} \neq \frac{1}{2}$, then $\frac{a}{b-2a} = \frac{c}{d-2c}$ to $\frac{MP}{AC} = \frac{DM}{DC}$.

Problem 5/3/27:

Note: Student solutions varied considerably. Award full credit (**5 points**) for any correct and complete solution. The rubric below is based on the official solution, with some guidance for how to award partial credit for students who submitted incorrect or incomplete solutions using other methods.

Award **2 points** if the student states and proves a useful general expression for $a_{n,k}$, such as

$$a_{n,k} = a_n + \binom{k}{1} a_{n+1} + \binom{k}{2} a_{n+2} + \cdots .$$

Award **1 point** of these 2 points if the student states the expression without proof, or if the proof is incomplete.

Award **3 points** if the student uses roots of unity or another method along with the preceding expression to show that if for all i and n, $|a_{i,n}| < M$, a contradiction emerges. Award **1 point** of partial credit for significant constructive progress towards this result, or **2 points** of partial credit if the solution is almost complete.

Award a total of **1 point** if the student calculates several values of $a_{n,k}$, which is a_n at time k, but makes no other constructive progress.

Award a total of **2 points** if the student states and proves a useful lemma. Graders have discretion to award additional credit if the student proves multiple lemmas or if the lemma is especially valuable. Award a total of **1 point** if the student states a useful lemma, but the proof is missing or incomplete. **Note:** the student doesn't have to call the key intermediate result a lemma to receive these 2 points.