Round 2 Grading Criteria

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/2/26:

5 points should be awarded for the correct configuration of numbers. No justification is required. Withhold a point for each number that is placed incorrectly.

If the configuration is not fully correct, you should specifically point out at least one error in your comments.

Problem 2/2/26:

2 points: Student recognizes that the inequality $ax + by \leq bx + cy$ is equivalent to the inequality $(b - c)y \leq (b - a)x$ and that the inequality $bx + cy \leq cx + ay$ is equivalent to the inequality $(b - c)x \leq (a - c)y$ (one point for each of the two inequalities).

1 point: Student recognizes that if $b > c$, then the left sides of the two inequalities ($(b - c)y \leq (b - a)x$ and $(b - c)x \leq (a - c)y$) are both positive, which means that the right sides of the two inequalities must also be positive, which gives us $b > a > c$.

1 point: Student recognizes that if $b > a > c$, then $ax > cx$ and $by > ay$, which gives us the inequality $ax + by > cx + ay$.

1 point: Student recognizes that the inequality $ax + by > cx + ay$ contradicts the inequality $ax + by \leq cx + ay$, and thus, $b \leq c$.

Note: Any student who completes some of the above steps should receive at least the number of points specified above for the corresponding steps. Additional points may be awarded for additional constructive progress towards the solution, even if the student's method differs from the official solution.

Problem 3/2/26:

4 points: Student recognizes (with justification) that the intersection of the interiors of P and Q is a square pyramid with the same base as P and Q, but with apex $\left(\frac{3}{2}, \frac{3}{2}, \frac{9}{4}\right)$. Regardless of which (correct) method is used, this part of the problem is worth four points. Partial credit may be awarded for constructive progress. For students who use the method in the official solution, the four points should be sub-divided as follows.

3 points: Student recognizes (with justification) that the intersection of P and Q is the set of points that satisfies $\frac{2z}{3} < x < 3 - \frac{2z}{3}$ and $\frac{2z}{3} < y < 3 - \frac{2z}{3}$. Award partial credit for constructive progress towards this result, and award at most two of the three points if the justification is missing or incomplete.

1 point: Student uses the above result to determine that the intersection of the interiors of P and Q is a square pyramid with the same base as P and Q, but with apex $\left(\frac{3}{2}, \frac{3}{2}, \frac{9}{4}\right)$.

1 point: Student uses the volume formula $\frac{1}{3}bh$ to find the volume, which is $\frac{27}{4} = 6.75$. If the student calculated the apex incorrectly, then they should still receive one point if they use the volume formula correctly.

Note: One point was deducted from the total score if a numerical approximation was used to calculate the volume.

Problem 4/2/26:

Note: Student solutions varied considerably.

1 point: Student recognizes that it is possible for a convex polyhedron to have one pivot point *and* provides an example (e.g., the center of a cube).

1 point: Student recognizes that in order to show that a convex polyhedron can have at most one pivot point, it is sufficient to show that a contradiction emerges if one considers the possibility that a convex polyhedron has a second pivot point.

3 points: Successful demonstration that a contradiction emerges if one considers the possibility that a convex polyhedron has a second pivot point. Partial credit (one or two points) may be given based on the quality of the student's constructive progress. To get one point of partial credit, it is sufficient to note that if a line through a pivot point contains two vertices, the vertices must be on opposite sides of the pivot point.

Problem 5/2/26:

Note: Student solutions varied considerably.

3 points: Student provides (with justification) a method of coloring all the positive integers with four colors. If the justification is missing or incomplete, but the coloring scheme is correct, two of three points should be awarded. If the student makes reasonable constructive progress, one of three points should be awarded.

2 points: Student shows (with justification) that it is impossible to color all the positive integers with three (or fewer) colors. Award one of the two points if the justification is missing or incomplete, or if the student makes a reasonable amount of constructive progress.