



USA Mathematical Talent Search

Round 1 Problems

Year 38 — Academic Year 2026-2027

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name, username, and USAMTS ID# on **every page you submit.**
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.
4. Submit your solutions by **October 13, 2026** via one (and only one!) of the methods below:
 - (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
Deadline: 10 PM Eastern / 7 PM Pacific on October 13, 2026.
 - (b) Mail: USAMTS
90 Broad Street
Suite 902
New York, NY 10004
Deadline: Solutions must be postmarked on or before October 13, 2026.
5. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the “Account” page.
6. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to “My Scores”. You will also receive an email when your scores and comments are available (provided that you did item #5 above).

**These are only part of the complete rules.
Please read the entire rules at www.usamts.org.**



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Each problem is worth 5 points.

1/1/38. Place two digits in each square to form a two-digit number. (Leading zeroes are allowed; for example, 01 could be in a square.)

- In each row and column, each digit from 0 to 9 must appear exactly once. All two-digit numbers in the grid must be different.
- A number will be prime if and only if it is circled.
- A given digit in a square must be one of the two digits in that square. (For example, if 8 is given in a square, the square might contain 18 or 81.)
- A number outside a row or column gives the unit digit of the sum of the five numbers in that row or column.

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

			1	5	5
6	1		3	2	4
2			0	1	2
4					3
9	4		8		8
7	4		6		



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2/1/38. Alice and Bob start writing down consecutive numbers at the same time, starting with the number “1”. They write numbers at the same rate, meaning that for all $n \geq 1$, Alice writes her n^{th} number at the same time that Bob writes his n^{th} number. Alice writes down Kaktovic numerals, which is a base-20 number system:

$$\underline{1}, \underline{2}, \dots, \underline{19}, \underline{1} \underline{0}, \underline{1} \underline{1}, \dots, \underline{19} \underline{19}, \dots$$

Bob writes down spreadsheet column codes, which follow a base-26 system with no zero:

$$A, B, C, \dots, Z, AA, AB, \dots, ZZ, \dots$$

Whenever Alice and Bob simultaneously write down a number with the same number of digits (in their respective number systems), they earn a *(20-26)-point*.

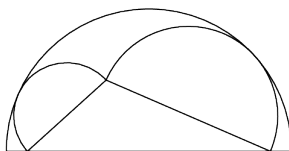
1. After Alice and Bob have each written down 2026 numbers, how many *(20-26)-points* do they have?
2. Prove that Alice and Bob can only ever earn finitely many *(20-26)-points*, and find the largest value n such that the n^{th} number Alice and Bob each write down earns a *(20-26)-point*.

3/1/38. There are an infinite number of lily pads, numbered $0, 1, 2, \dots$ from left to right. A frog is initially on lily pad 0. There exist fixed probabilities $q_1, q_2, q_3, \dots$, so that at any point, the probability that the frog leaps n lily pads to the right is q_n . (Also, $q_1 + q_2 + q_3 + \dots = 1$, so the frog is always leaping to the right.)

It turns out that the probability that the frog lands on lily pad n is p , for any positive integer n .

1. Compute q_n in terms of p .
2. Compute the expected number of leaps it takes for the frog to either land on or pass lily pad n .

4/1/38. Two semicircles, with radii a and b , are inscribed in a semicircle of radius r , as shown below. Prove that $a + b = r$.





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5/1/38. Let $n \geq 3$ be a positive integer. A circle is marked with n distinguishable points, and each point is labeled with a positive integer. The sum of all n labels is $2n + 3$, and there is no way to draw an arc on the circle such that the points on the arc have labels summing to $n + 1$. In terms of n , in how many ways can the labels be chosen? (Since the points are distinguishable, rotations and reflections are distinct.)

For example, if $n = 3$, the numbers could be 7, 1, and 1, which gives arcs with sums 0, 1, 2, 7, 8, and 9.

Problems by Aidan Deshong and USAMTS Staff.

Round 1 Solutions must be submitted by **October 13, 2026**.

Please visit <https://www.usamts.org> for details about solution submission.

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