USA Mathematical Talent Search

Round 3 Problems

Year 37 — Academic Year 2025-2026

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name, username, and USAMTS ID# on **every page you submit**.
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.
4. Submit your solutions by **January 5, 2026** via one (and only one!) of the methods below:
   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 10 PM Eastern / 7 PM Pacific on January 5, 2026.
   (b) Mail: USAMTS
   90 Broad Street
   Suite 902
   New York, NY 10004
   Deadline: Solutions must be postmarked on or before January 5, 2026.
5. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the "Account" page.
6. Round 3 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My Scores". You will also receive an email when your scores and comments are available (provided that you did item #5 above).

These are only part of the complete rules.
Please read the entire rules at www.usamts.org.

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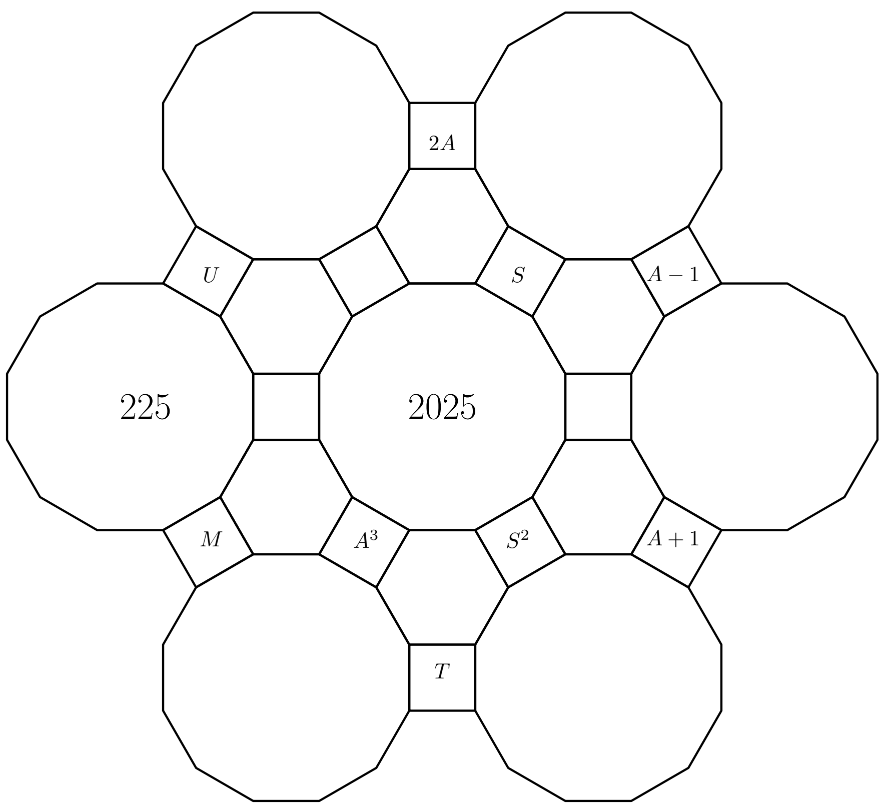
Each problem is worth 5 points.

1/3/37. A magical wizard has given you a formidable challenge. Below is a section of a 4−6−12 tiling of the plane. You must place a positive integer in each square, hexagon, and dodecagon such that:

- The only integer that is repeated is 1.
- The value in each dodecagon is the product of the values in the squares connected to it.
- The value in each hexagon is the least common multiple of the values in the three squares connected to it.
- $U + S + A + M + T + S = 57$.

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)



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2/3/37. The game of *summing solitaire* is played as follows. A deck of 101 cards (numbered 1 through 101) is randomly shuffled. The cards are drawn one at a time. As each card is drawn, it is put into the *scoring pile* if its number is larger than the numbers on all other cards that have been drawn so far, and otherwise it is discarded. (The first card drawn is placed in the scoring pile.) After drawing all 101 cards, the final score is the sum of all the numbers on the cards in the scoring pile.

What is the probability that the final score is even?

3/3/37. Prove that for any positive integer m, there exists a set S of $2m$ consecutive positive integers with the following property: for all nonnegative integers k, n, if S contains one of $\{3^k(3n+1), 3^k(3n+2)\}$, then it also contains the other.

4/3/37. Let O be the circumcenter of triangle ABC, and let H be the orthocenter. Let O_A be the circumcenter of triangle BOC, and define O_B, O_C similarly. Let H_A be the circumcenter of triangle OO_BO_C, and define H_B, H_C similarly. Prove that AH_A, BH_B, CH_C, and HO are concurrent.

5/3/37. Let $N > 1$ be an integer, and suppose that $N = \prod_{i=1}^{n} p_i^{e_i}$ is the factorization of N into distinct prime factors $p_1, p_2, \ldots, p_n$. Assume that $p_1 > p_2 > \cdots > p_n$ and define

$$A(N) = \frac{\sum_{i=1}^{n} e_i \cdot p_i}{\sum_{i=1}^{n} e_i}.$$

Further define

$$B(N) = p_1 - \cfrac{1}{p_1 - \cfrac{1}{p_1 - \cfrac{1}{\ddots - \cfrac{1}{p_2 - \cfrac{1}{\ddots - \frac{1}{p_n}}}}}},$$

where p_1 occurs e_1 times in the continued fraction, p_2 occurs e_2 times, and so on. For example,

$$B(3^3 \cdot 5^2) = 5 - \cfrac{1}{5 - \cfrac{1}{3 - \cfrac{1}{3 - \cfrac{1}{3}}}}$$

Find all positive integers N so that $A(N) = B(N)$.

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Problems by Tanny Libman and USAMTS Staff.
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Please visit **https://www.usamts.org** for details about solution submission.